

Introduction

Polarization effects due to dielectric mismatch between two media are known to influence the spatial distribution of the charged particles in the vicinity of the interface. Few open-source software packages support coarse-grained models with surface polarization effects.

We implement and extend three methods for incorporating surface polarization effects into LAMMPS: the boundary element method (BEM) for solving the Poisson's equation, the image charge computation (ICC*) method, and the direct optimization of an energy functional of induced charge density. Our implementation is complementary to COPPS and works seamlessly with SSAGES.

Methods and Algorithms

Gauss's law: $\nabla \cdot \mathbf{E}(\mathbf{r}) = \frac{\rho_f(\mathbf{r})}{\epsilon_0 \epsilon}$

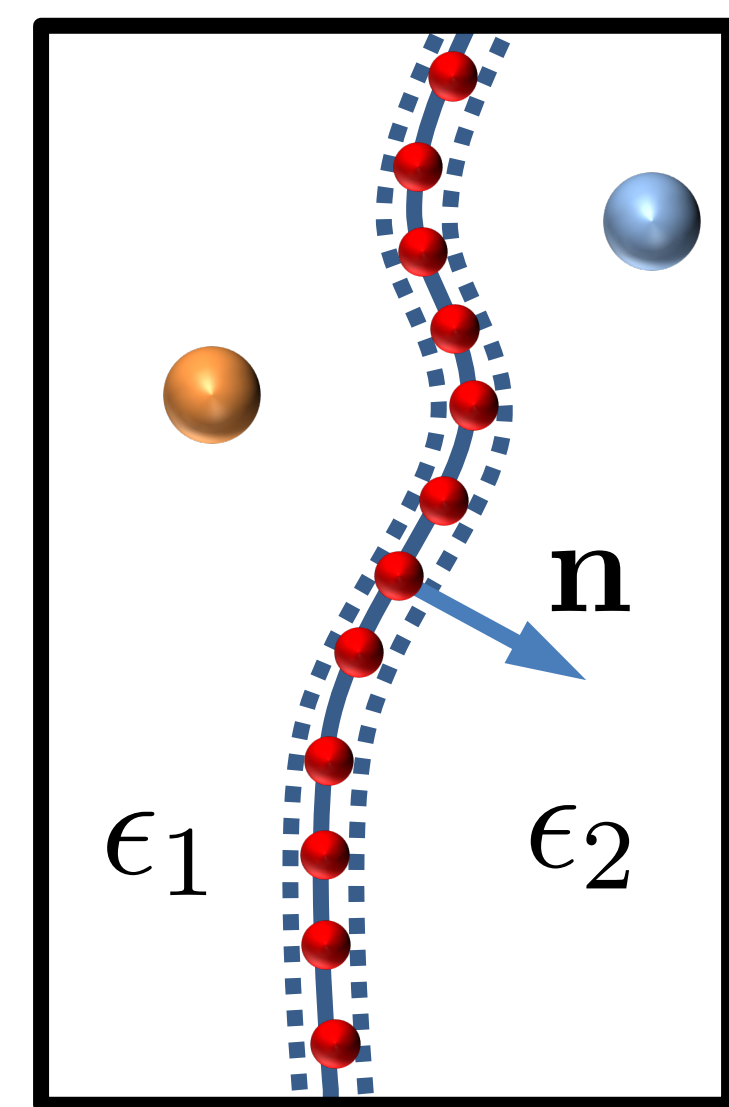
Displacement equation:

$$\rho_f(\mathbf{r}) = \epsilon_0 \nabla \cdot [\epsilon(\mathbf{r}) \mathbf{E}(\mathbf{r})] = \nabla \cdot \mathbf{D}(\mathbf{r})$$

Reduction to interface:

$$\epsilon_0 [\epsilon_2 \mathbf{E}^{(2)}(\mathbf{s}) - \epsilon_1 \mathbf{E}^{(1)}(\mathbf{s})] \cdot \mathbf{n}(\mathbf{s}) = \sigma_f(\mathbf{s})$$

$$\epsilon_0 [\mathbf{E}^{(2)}(\mathbf{s}) - \mathbf{E}^{(1)}(\mathbf{s})] \cdot \mathbf{n}(\mathbf{s}) = \sigma_f(\mathbf{s}) + \sigma_b(\mathbf{s})$$



$$\bar{\epsilon} = (\epsilon_1 + \epsilon_2)/2$$

$$\Delta\epsilon = \epsilon_2 - \epsilon_1$$

1. Boundary element method (BEM) – COPPS

$$\bar{\epsilon} \sigma_b + \epsilon_0 (\Delta\epsilon) \mathbf{E}_b \cdot \mathbf{n} = (1 - \bar{\epsilon}) \sigma_f - \epsilon_0 (\Delta\epsilon) \mathbf{E}_f \cdot \mathbf{n}$$

- GMRES to solve a set of linear equations iteratively for σ_b

2. Image charge computation (ICC*) – Holm et al.

$$\sigma_b^{(m+1)} = (1 - \omega) \sigma_b^{(m)} + \omega \left[\frac{1 - \bar{\epsilon}}{\bar{\epsilon}} \sigma_f(\mathbf{s}) - \epsilon_0 \frac{\Delta\epsilon}{\bar{\epsilon}} \mathbf{E}^{(m)} \cdot \mathbf{n} \right]$$

- SOR to solve a set of linear equations for σ_b

3. Optimization of an energy functional of induced surface charge density – Olvera et al.

$$I[\rho_f; \sigma_b] = \frac{1}{2} \iint \rho_f(\mathbf{r}) R_{\rho\rho}(\mathbf{r}, \mathbf{r}') \rho_f(\mathbf{r}') d\mathbf{r} d\mathbf{r}' + \frac{1}{2} \iint \rho_f(\mathbf{r}) R_{\rho\sigma}(\mathbf{r}, \mathbf{s}) \sigma_b(\mathbf{s}) d\mathbf{r} d\mathbf{s} + \frac{1}{2} \iint \sigma_b(\mathbf{s}) R_{\sigma\sigma}(\mathbf{s}, \mathbf{s}') \sigma_b(\mathbf{s}') d\mathbf{s} d\mathbf{s}'$$

- Matrix inversion to solve a set of linear equations for σ_b

Implementation and Validation

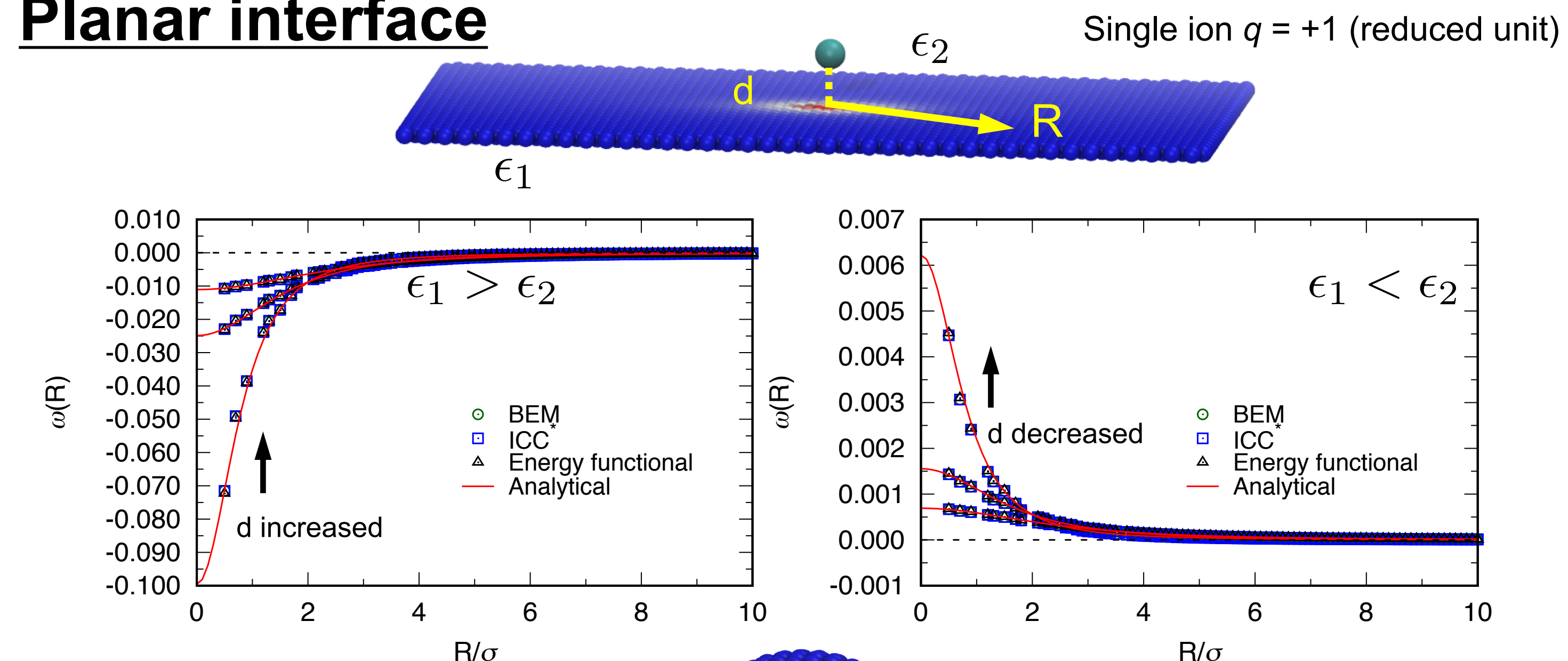
A new user package in LAMMPS: USER-DIELECTRIC

- new atom style: **AtomVecDielectric**
- new *pair* and *k-space* styles for per-atom electrical fields
- new *fixes*: **FixPolarizeBEM**, **FixPolarizeICC**, **FixPolarizeFunctional**

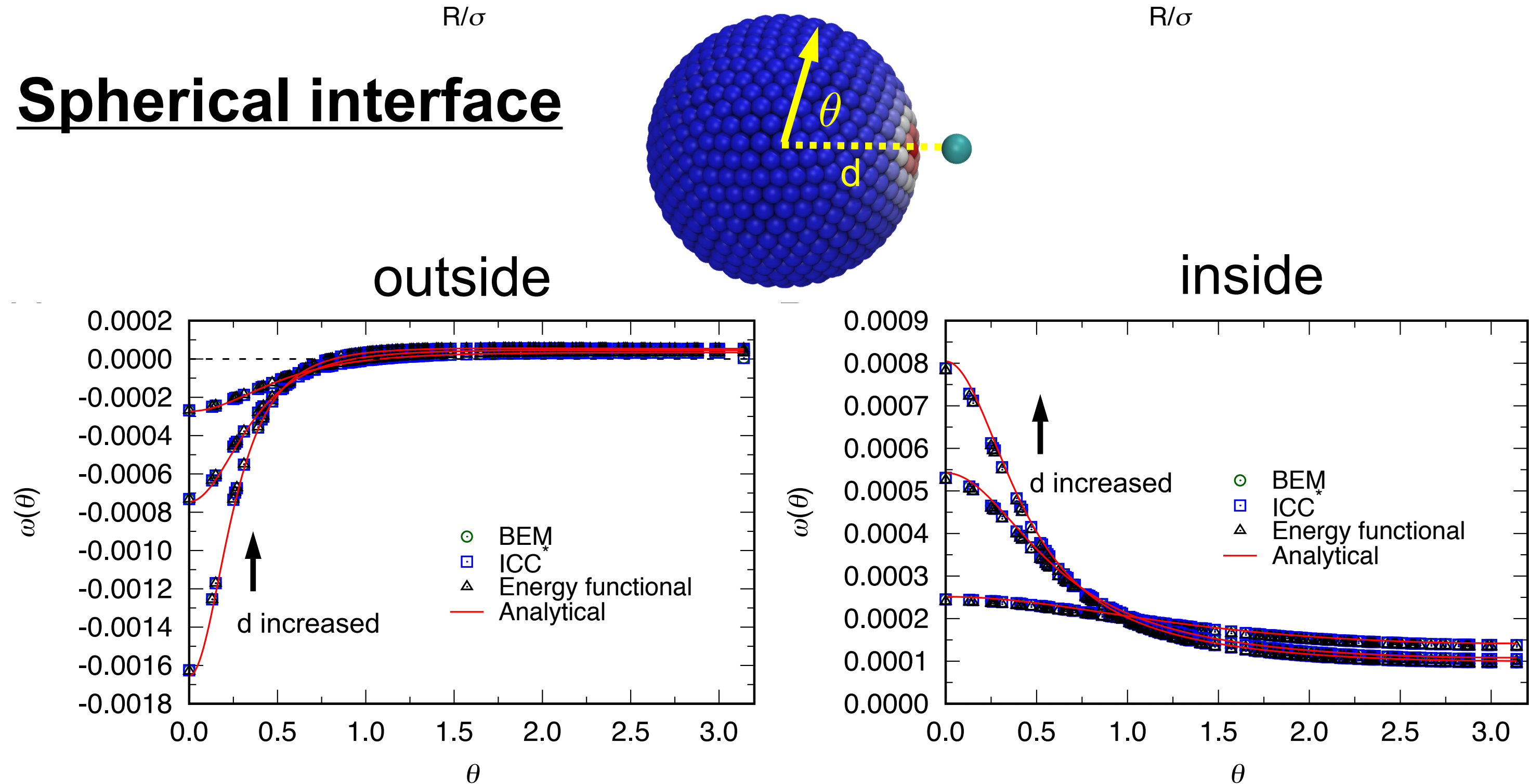
Advantages:

- Compatible with existing functionalities in LAMMPS including molecular, rigid bodies, time integrators, long-range electrostatics solvers (PPPM and MSM), and built-in analysis
- Support moving and/or flexible interfaces
- Compatible with SSAGES out of the box

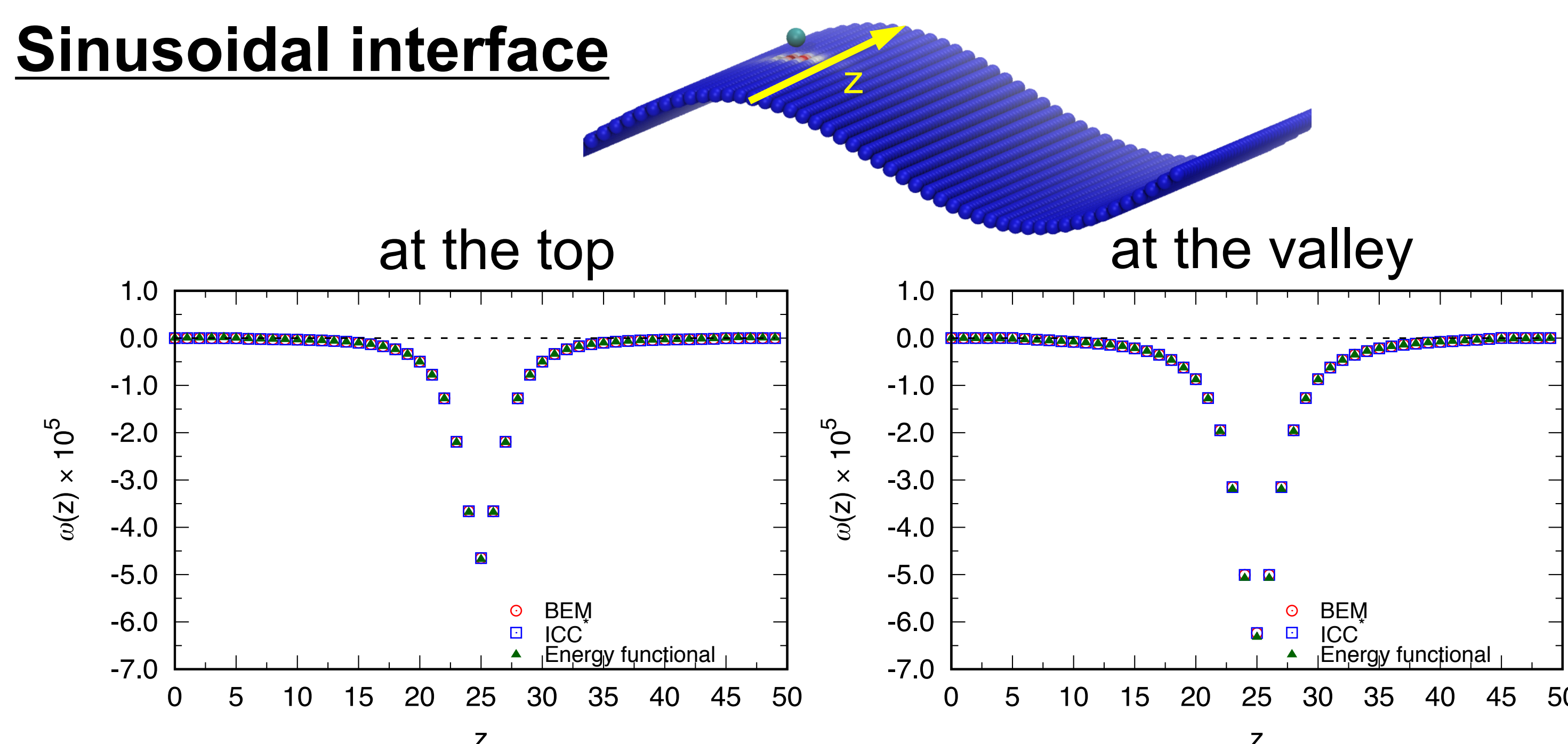
Planar interface



Spherical interface



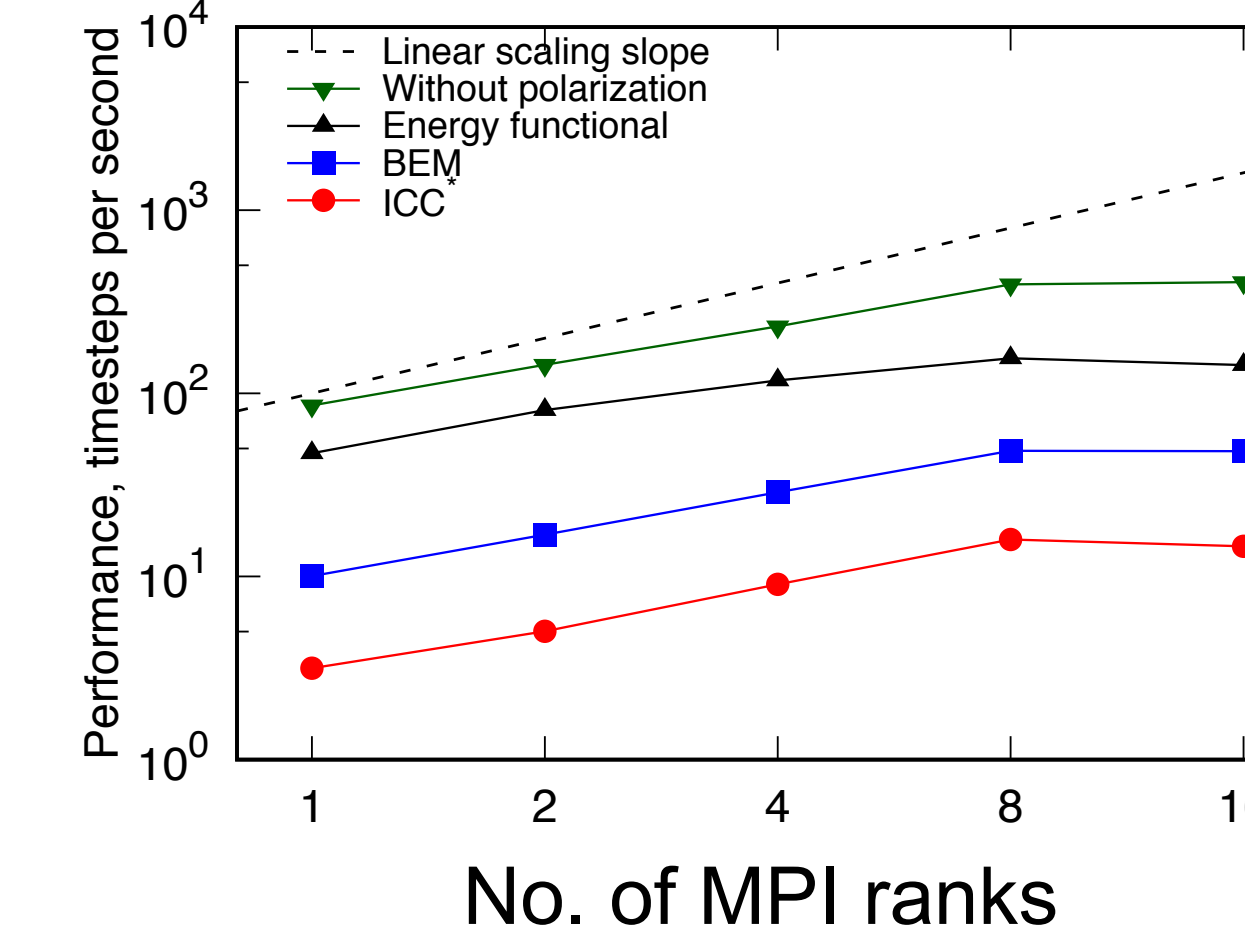
Sinusoidal interface



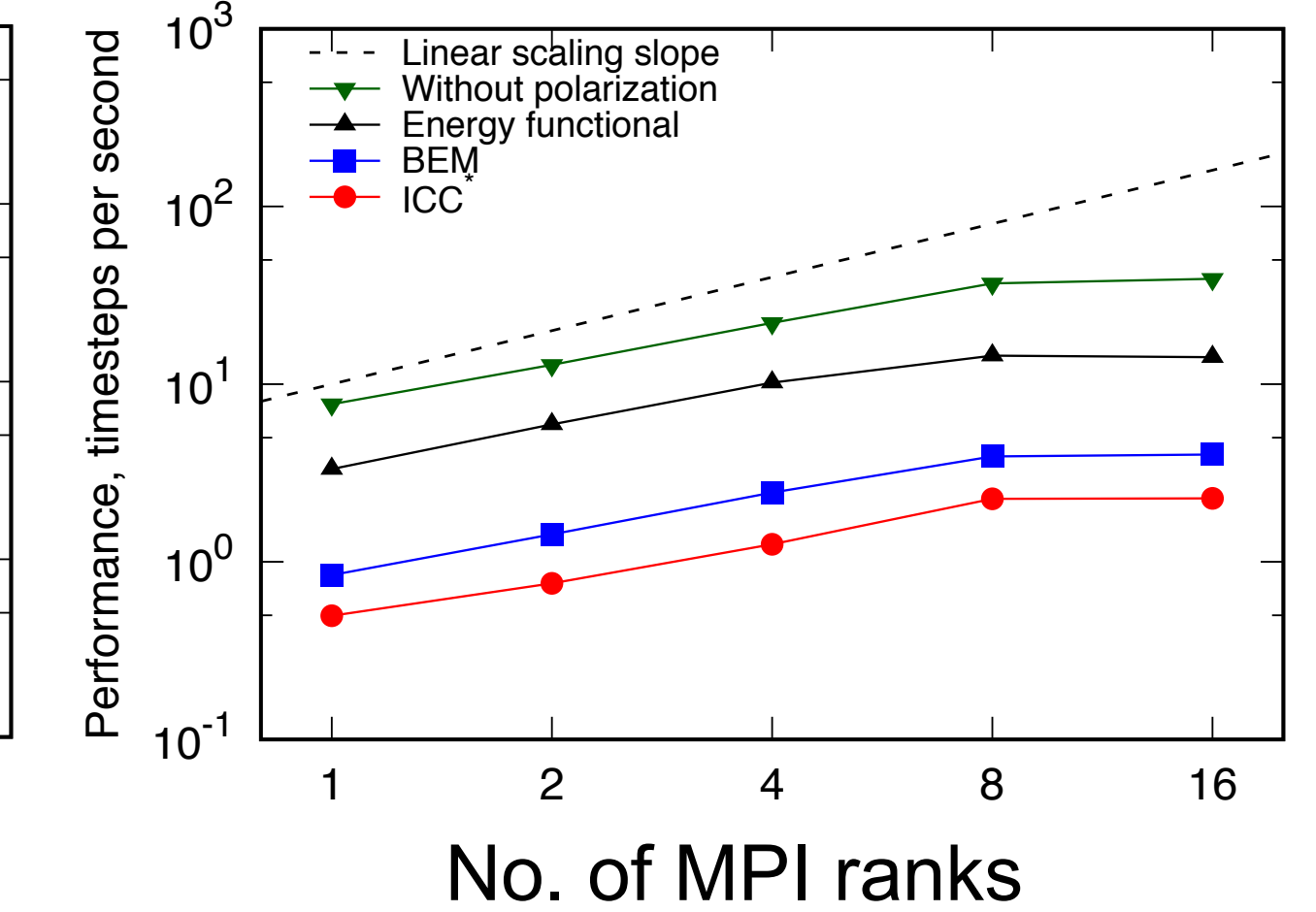
Simulation Performance

Benchmark system: Confined polyelectrolyte in a spherical cavity

No. of surface patches = 642



No. of surface patches = 2562



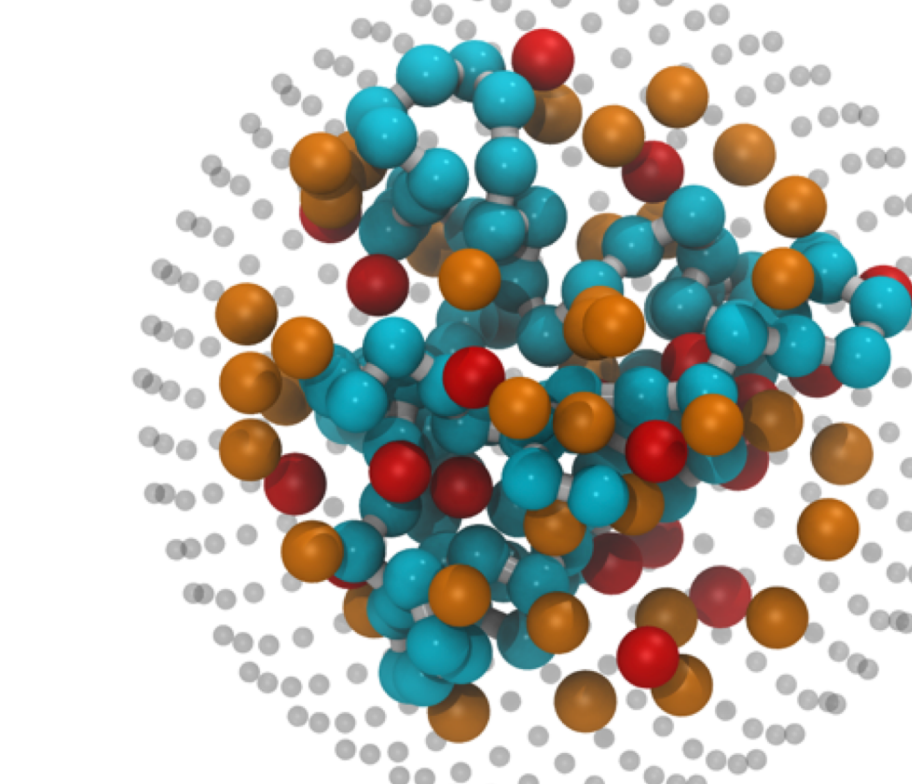
Strong scaling on a single node: built with LAMMPS version 16Mar18

Case study: Confined Polyelectrolyte

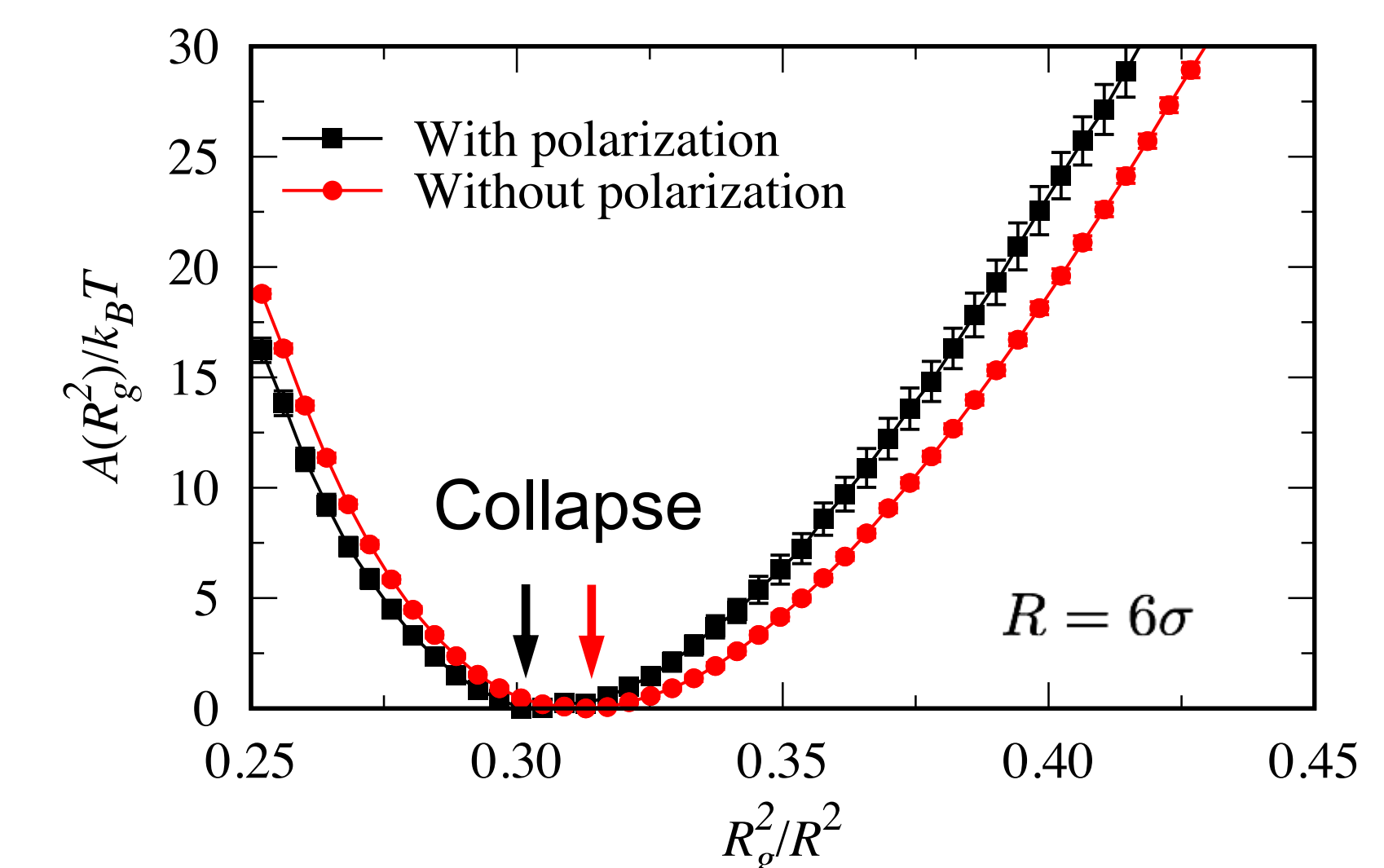
Free energy profile of the polyelectrolyte radius of gyration using umbrella sampling with SSAGES

$l_B/\sigma = 4.0$ $\epsilon_{in}/\epsilon_{out} = 10$

100-mer + 30 trivalent counterions + 10 monovalent ions



100-mer polyelectrolyte (cyan)
M trivalent counterions (red)
N monovalent counterions (orange)
spherical interface: induced charges (gray)

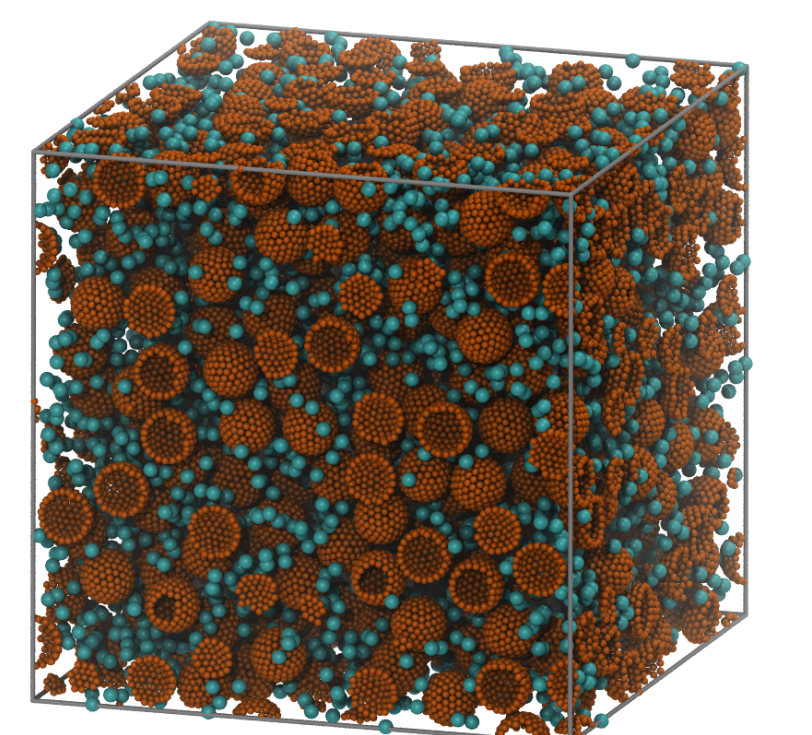


Key findings:

- Polarization effects leads to more compact conformation in the collapse state.
- Trivalent counterions increase the free energy barrier between collapse and coil states.

Ongoing and future work

- Validation with COPPS
- Collaboration with Juan de Pablo and Whitmer groups using SSAGES for efficient sampling and free energy calculations
- Collaboration with Talapin group for nanoparticle assembly studies
- Application to other case studies including polyelectrolytes, salt-doped block copolymers, and proteins



Metallic/Dielectric particle assembly

References and Acknowledgements

Acknowledgments: SSAGES for umbrella sampling; WHAM code by Alan Grossfield (U Rochester)

References

- Luijten *et al.* *Phys. Rev. Lett.* **2014**, 113, 017801 (BEM); Holm *et al.* *J. Chem. Phys.* **2010**, 132, 154112 (ICC*); Olvera de la Cruz *et al.* *Phys. Rev. Lett.* **2017**, 119, 138002. (Energy functional)

Funding: MICCOM