

**Oct 13-14, 2022 - Argonne National Lab**

# **MICCOM Workshop & Hands-on tutorials**

Argonne  
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# Computational Spectroscopy

# Electronic structure calculations at zero temperature

**Density functional theory (DFT)**

**Many body perturbation theory**

**Quantum embedding (QDET)**

**Spin Hamiltonians**

**Advanced Sampling**

**First Principles MD**

**Free energies  
Non-equilibrium processes**

**Thermodynamic properties & vibrational spectroscopies**

**Electronic and optical spectroscopies**

**Spin and coherent spectroscopies**



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**First Principles MD**

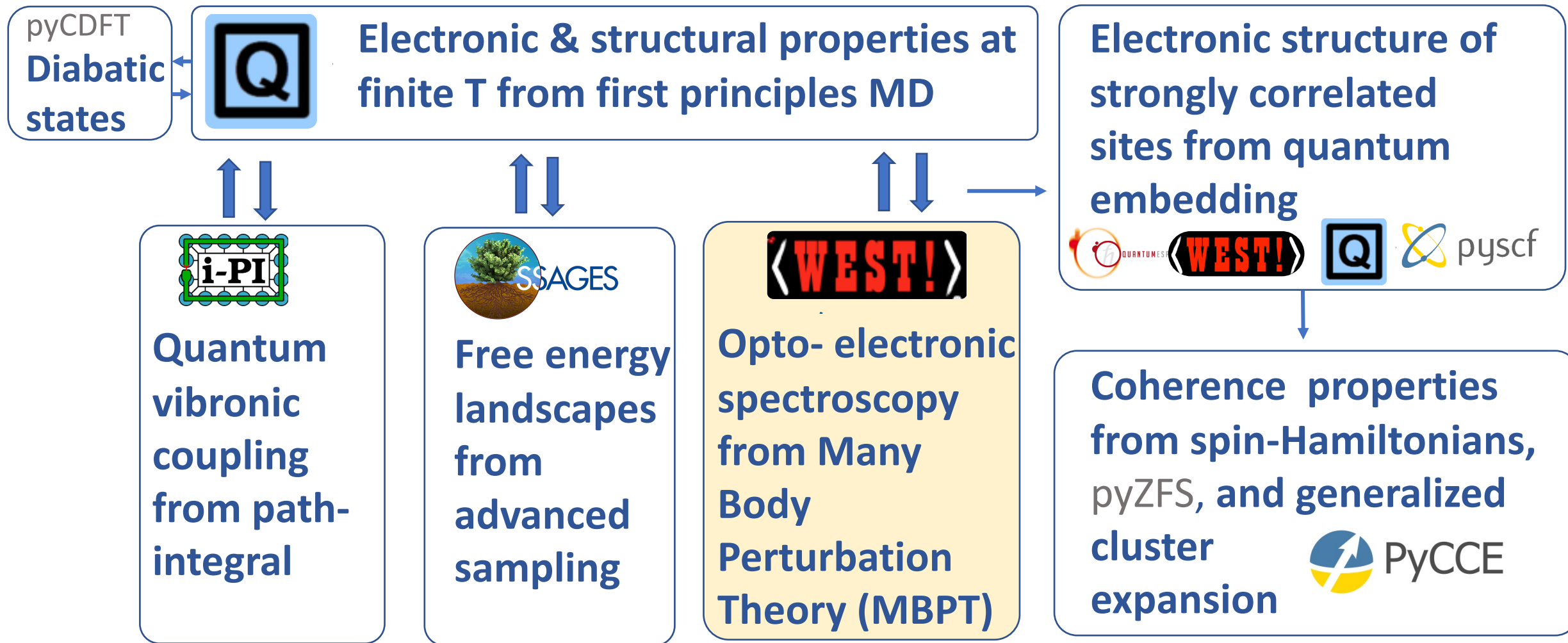
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**Electronic and optical spectroscopies**

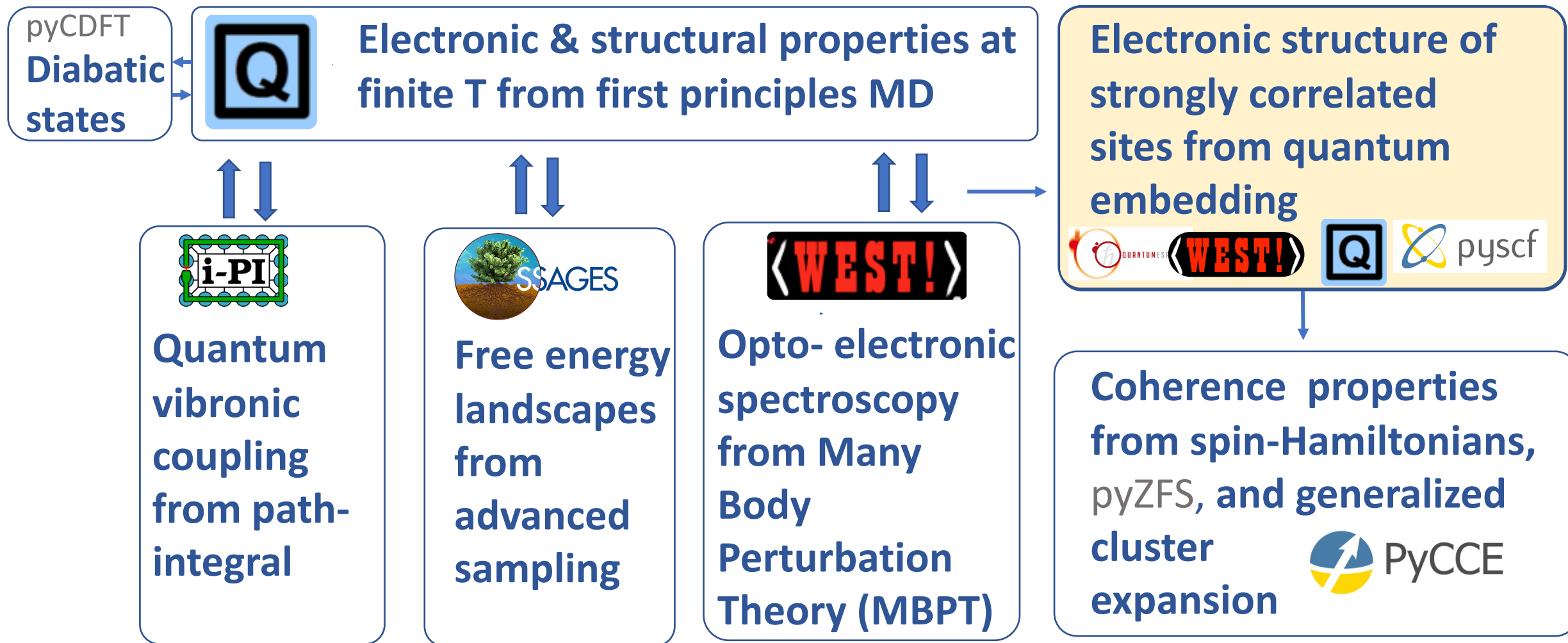
**Spin and coherent spectroscopies**

# A modular strategy for innovation

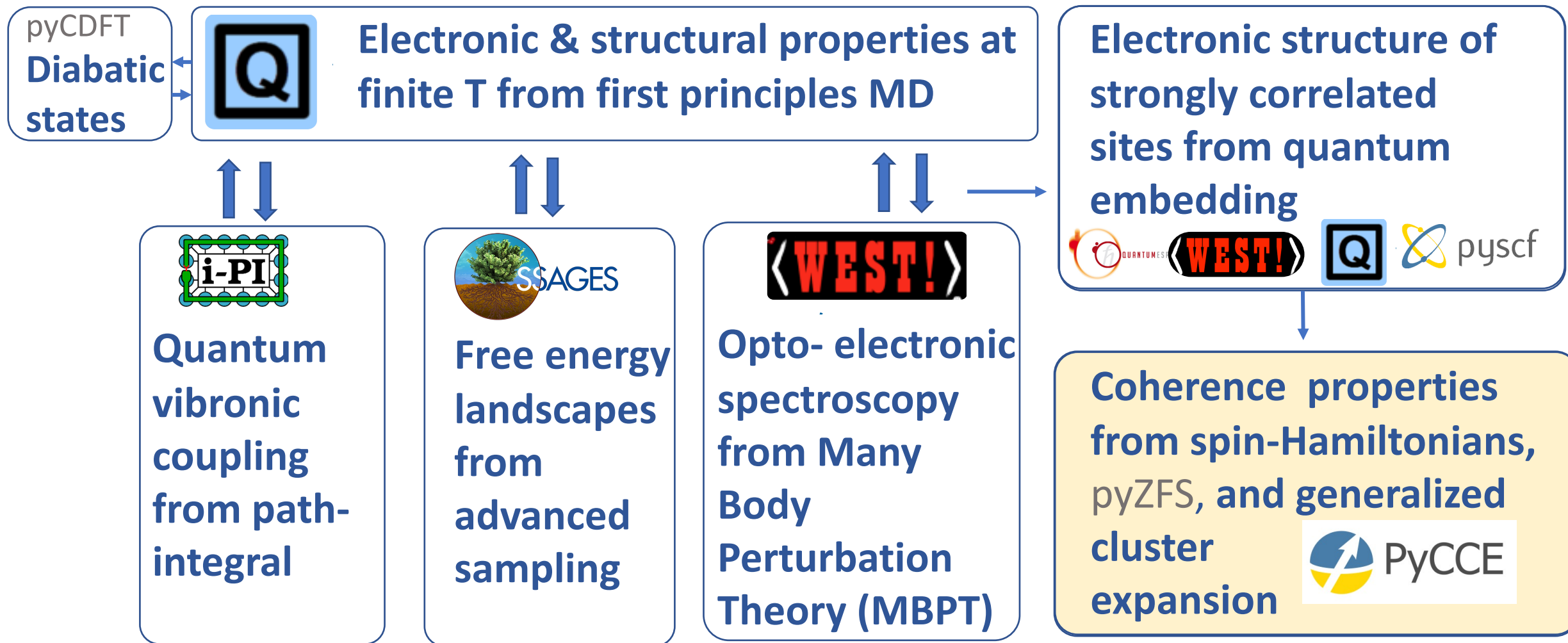




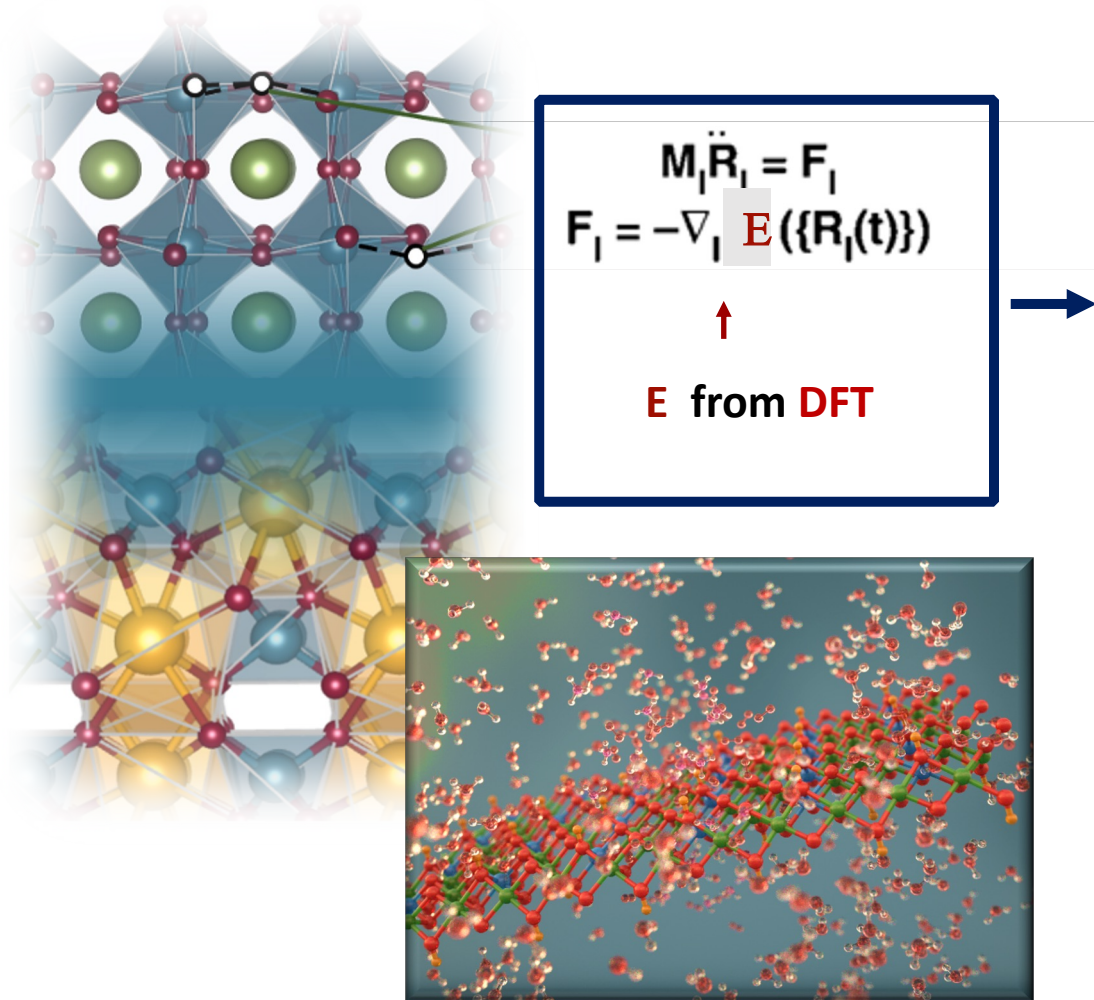
# A modular strategy for innovation



# A modular strategy for innovation



# Structural models derived with the aid of DFT and first principles molecular dynamics (FPMD)



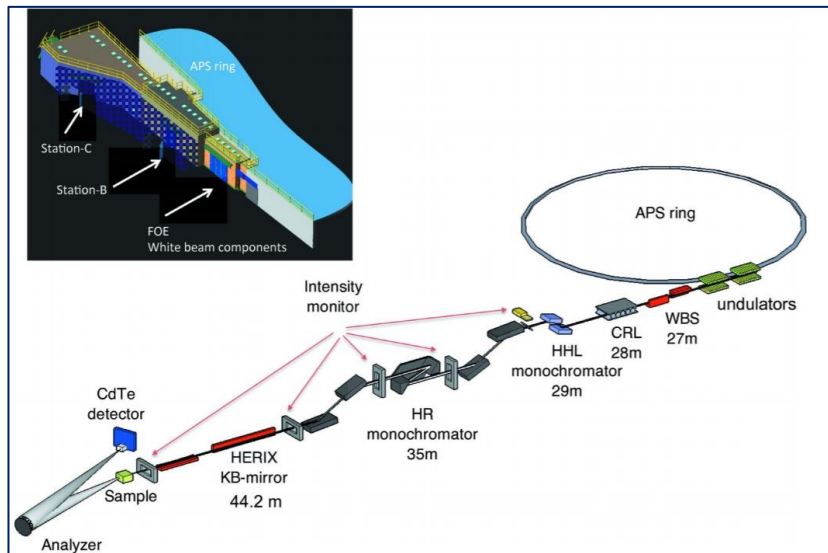
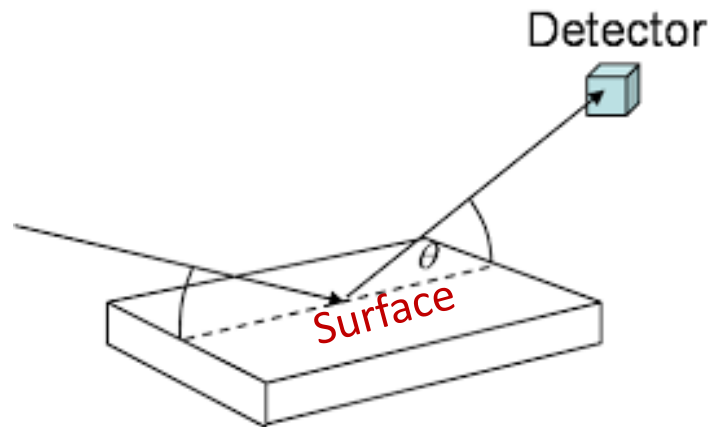
## Spectroscopic signatures

### Computational spectroscopy is key

- To validate atomistic structural models by comparing with experiments
- To understand & predict light-matter & external field interaction processes

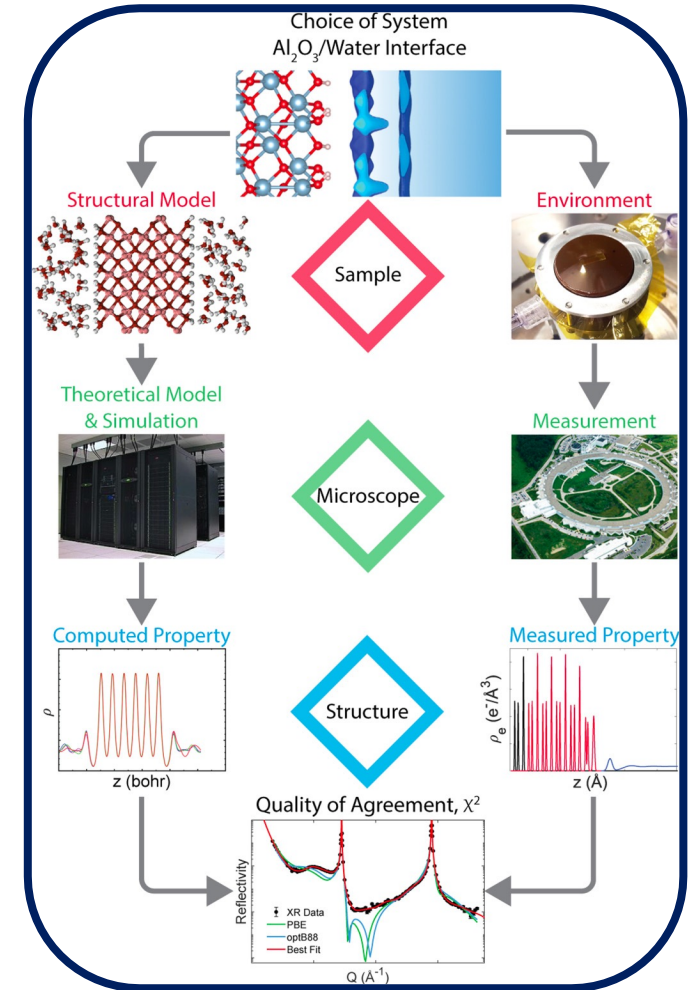


# Structural models derived with the aid of DFT and first principles molecular dynamics

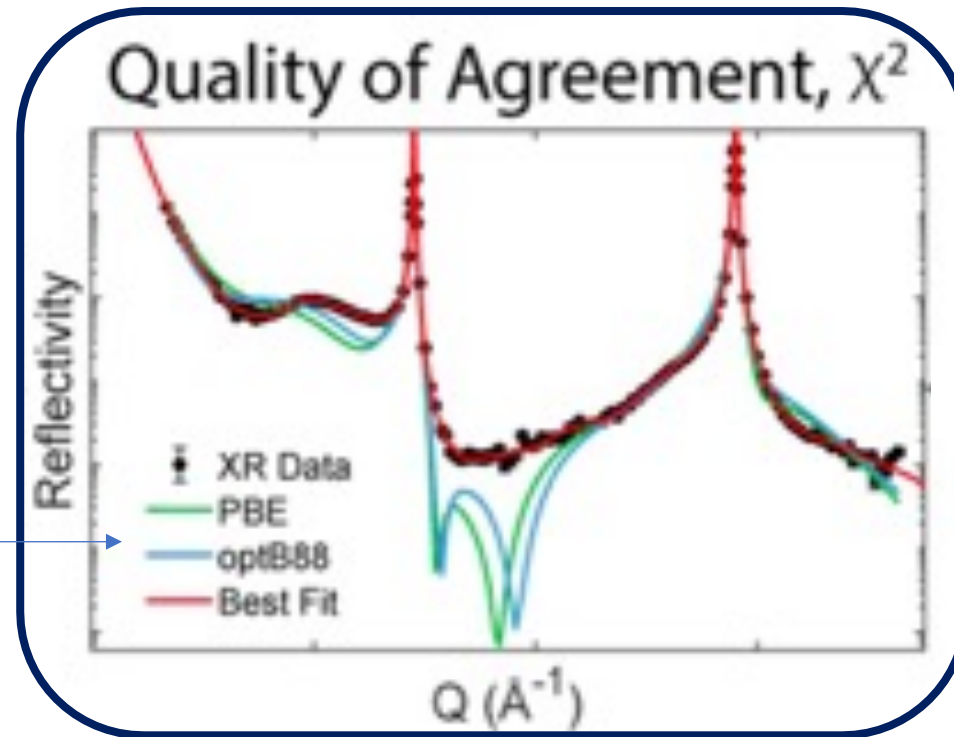


Experimental validation of structural models

Validation of FPMD of oxide/water interfaces with X-ray reflectivity data

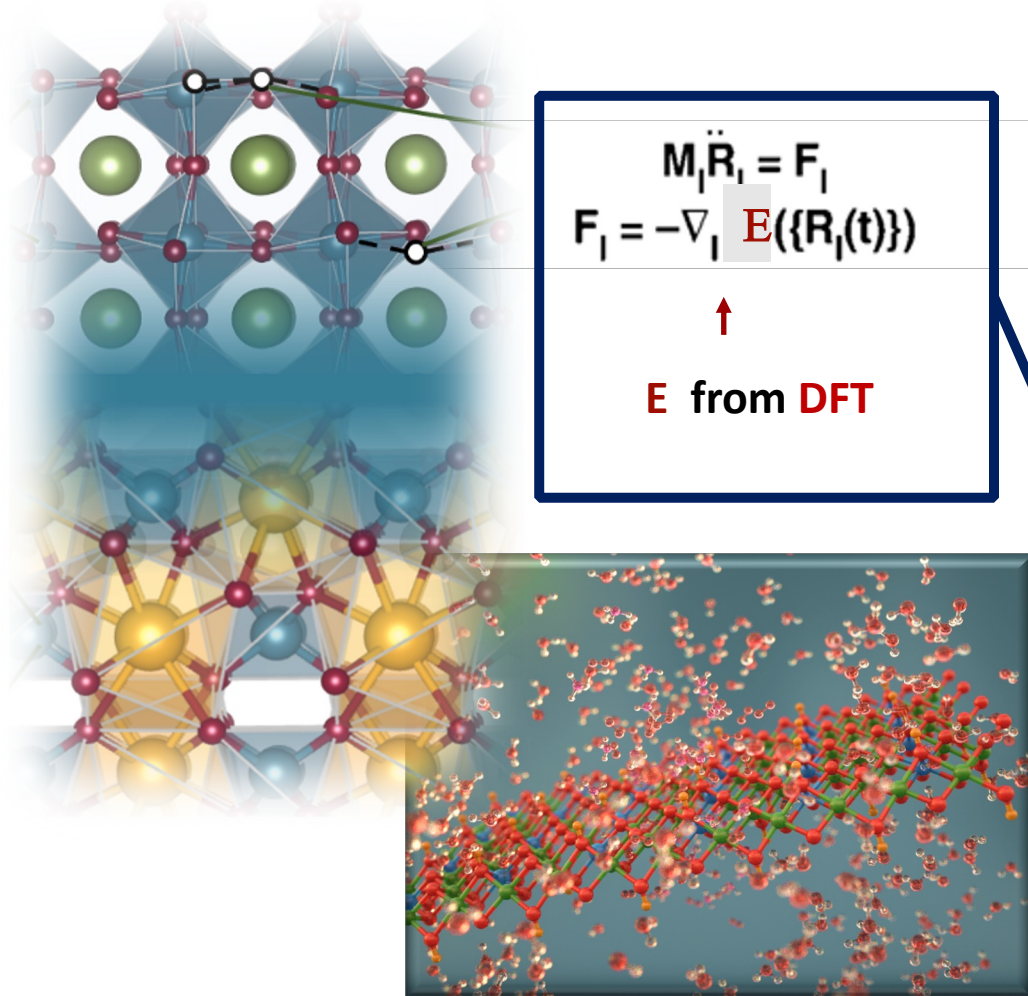


# Structural models derived with the aid of DFT and first principles molecular dynamics



Different density functionals

# Structural models derived with the aid of DFT and first principles molecular dynamics (FPMD)



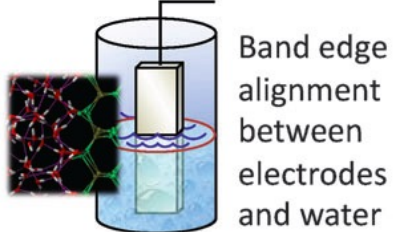
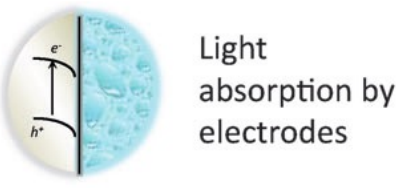
## Spectroscopic signatures

### Computational spectroscopy is key

- To validate atomistic structural models by comparing with experiments
- To understand & predict light-matter & external field interaction processes

# Spectroscopy on MD samples using **Many Body Perturbation Theory**

- Development of post-DFT **theories** for spectroscopic characterization of materials is key to prediction and design of new systems & properties

Processes of interest	 <p>Band edge alignment between electrodes and water</p>	 <p>Light absorption by electrodes</p>
Experiments	<p>Photoemission spectroscopy <b>GW</b></p>	<p>UV-vis and ellipsometry spectroscopy <b>BSE</b></p>

## Spectroscopic Characterization

### Many Body Perturbation Theory

- Photoemission
- Absorption
- X-ray photoemission
- Photoluminescence
- Deep level transient spectroscopy
- Ultrafast spectroscopy
- Non-radiative recombination

# Computational Spectroscopy

**Why is it useful?**

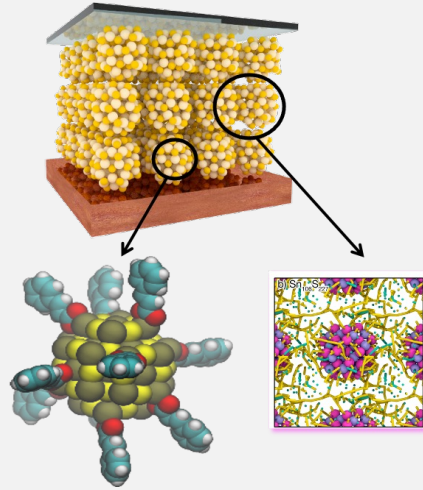
**Why is it interesting?**

**Which problems does it help us solve?**

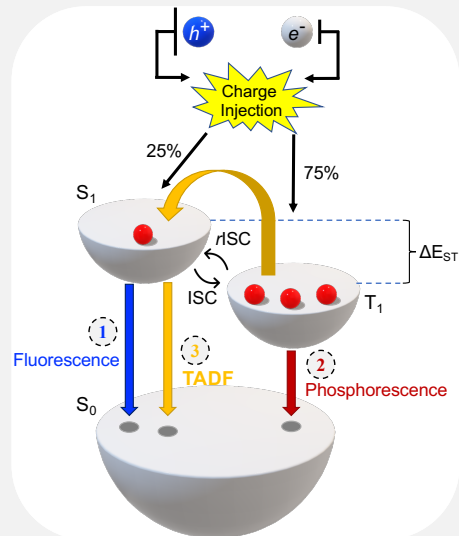
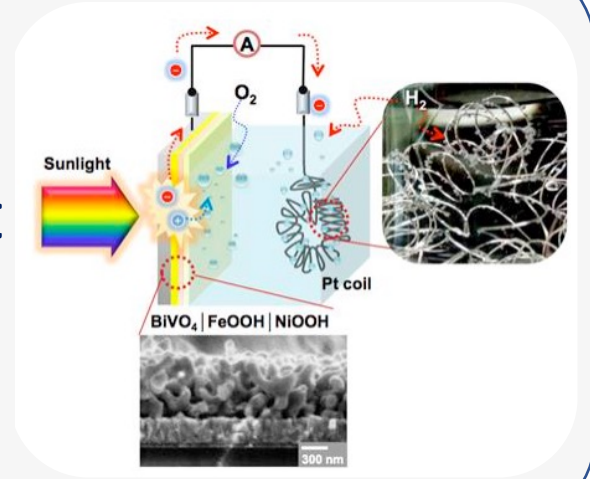


# Multi-faceted processes and complex materials

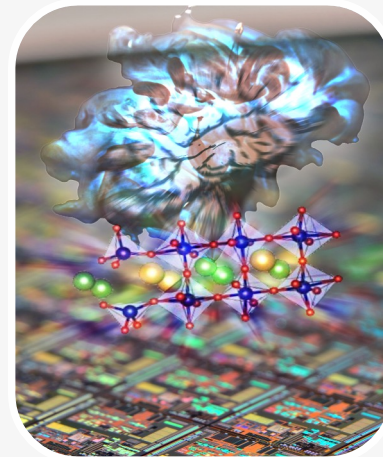
Can we design easy to make and cheap **solar cells** using colloidal quantum dots?



How can we trigger desired photo-reactions to **generate clean fuel** at the interface of photo-absorbers, catalysts and water?

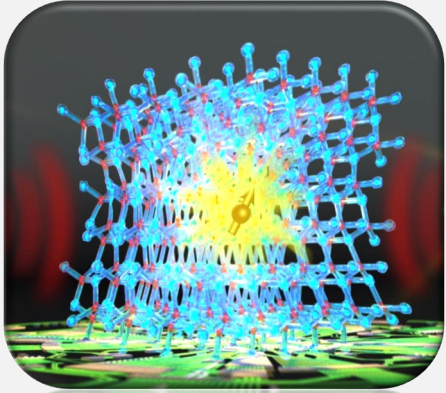


How do we design efficient **all organic** light emitting diodes (OLEDs)?



Which materials are suitable for energy-efficient **neuromorphic platforms** and **low power electronics**?

# Multi-faceted processes and complex materials



Can we predict solid state and molecular systems to realize **spin-qubit for quantum technologies?**



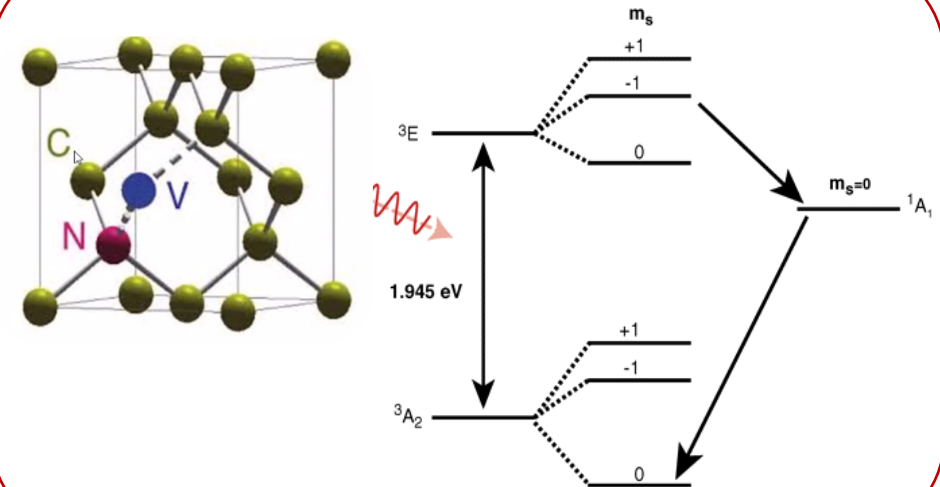
Harness the power of **quantum information:**

- Computing
- Sensors
- Communications

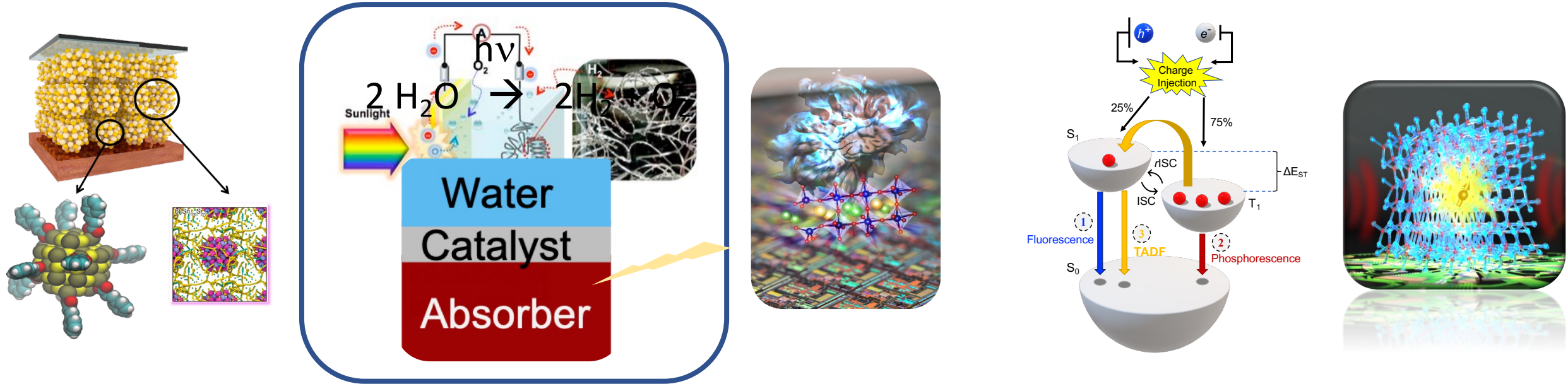
- Use electronic states of defects as **two-level systems**

- Optically address the two-level systems

\*Optical transition from ground to an excited state, followed by a spin-selective decay path with nonradiative transitions between states of differing spin multiplicity



# Multi-faceted processes and complex materials

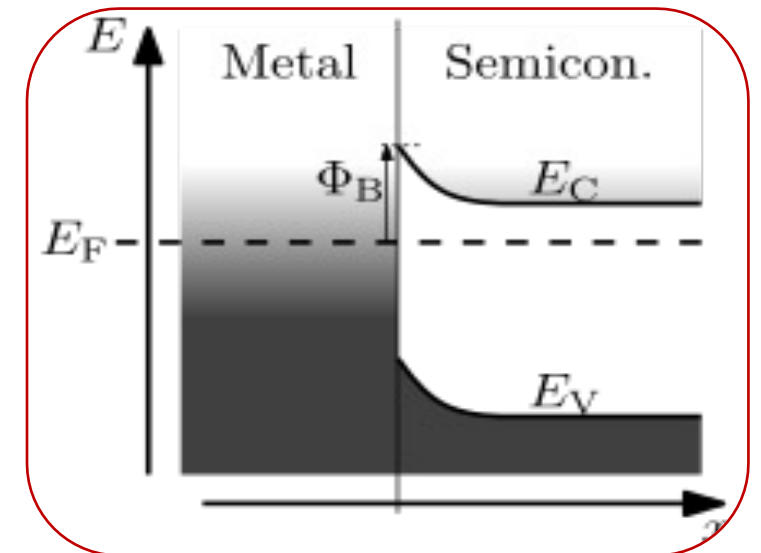
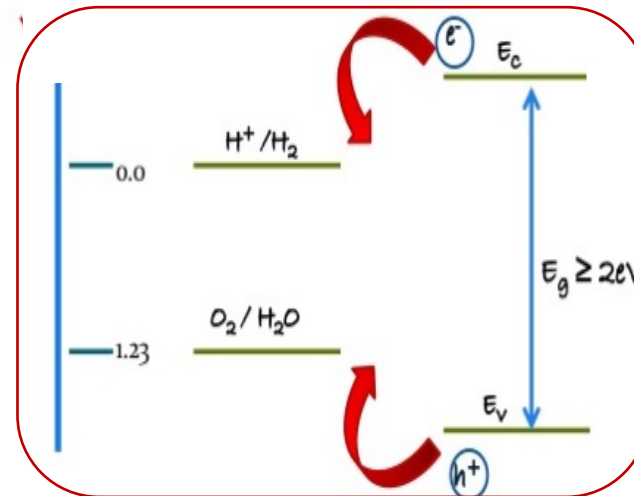
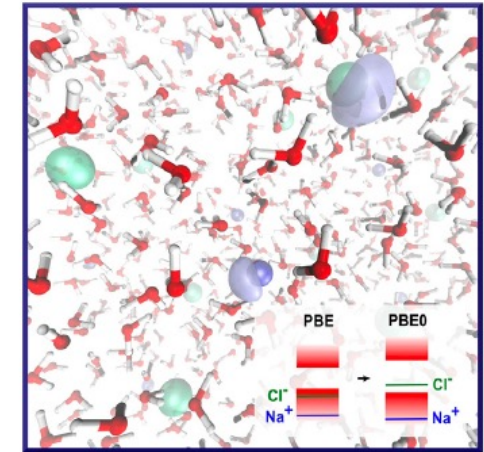
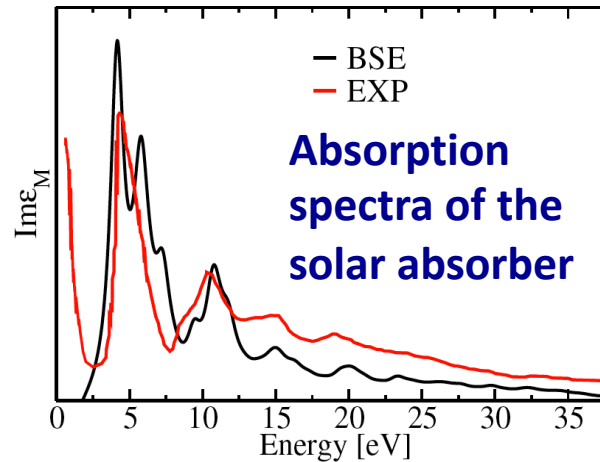


- Heterogeneous systems with **interfaces**, **defects** and complex **building blocks**
- Interaction of matter & **light** and matter & **external fields**

# A difficult problem with many components

## Simulations and predictions of numerous properties

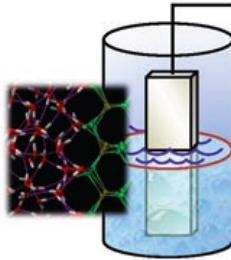
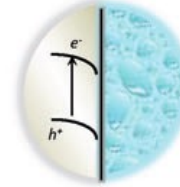
- Reasonable model of (salty) **water** (FPMD)
- Atomistic model of solid/liquid interfaces (FPMD) & their electronic properties ( **band offsets** and **Schottky barriers** )
- **Charge transport** @interfaces





# Spectroscopy on MD samples

Use first principles MD trajectories and compute **electronic properties** from **many body perturbation** theory → Electronic properties at **finite T** w/statistical errors

Processes of interest	 <p>Band edge alignment between electrodes and water</p>	 <p>Light absorption by electrodes</p>
Experiments	Photoemission spectroscopy <b>GW</b>	UV-vis and ellipsometry spectroscopy <b>BSE</b>
Theory	Single particle Green's functions $G^{(1)}$ : poles represent energies to add or remove an electron from a solid or molecule	Two particle correlation function $L$ : poles correspond to neutral excitation energies of interacting electrons
Equations	<b>Dyson's equation</b> : relates $G^{(1)}$ to self energy $\Sigma$ (effective potential of interacting electrons)	<b>Bethe Salpeter equation</b> : relates $L$ to the variation of $\Sigma$ with respect to $G^{(1)}$



# Solution of the Hedin's equations

$$\left(\hat{T} + \hat{V}_{ion} + \hat{V}_H + \hat{V}_{xc}\right) |\psi_n\rangle = \varepsilon_n |\psi_n\rangle \quad \text{DFT}$$

$$\left(\hat{T} + \hat{V}_{ion} + \hat{V}_H + \hat{\Sigma}(E_n^{QP})\right) |\psi_n^{QP}\rangle = E_n^{QP} |\psi_n^{QP}\rangle \quad \text{MBPT}$$

$\Sigma$  expressed in terms of the **dynamically screened Coulomb potential**

$$\Sigma(\mathbf{r}, \mathbf{r}'; \omega) = \int \frac{d\omega'}{2\pi} G(\mathbf{r}, \mathbf{r}'; \omega + \omega') W(\mathbf{r}, \mathbf{r}'; \omega')$$

$$G(\mathbf{r}, \mathbf{r}'; \omega) = \langle \mathbf{r} | \frac{1}{\omega - H} | \mathbf{r}' \rangle$$

$$W(\mathbf{r}, \mathbf{r}'; \omega) = \int d\mathbf{r}'' \epsilon^{-1}(\mathbf{r}, \mathbf{r}''; \omega) v_c(\mathbf{r}'', \mathbf{r}')$$

Use an expression in terms of  
of  
Kohn-Sham electronic  
states from density-density  
response functions  
or  
From DFT calculations in  
electric field

$$E_n^{QP} = \varepsilon_n^{KS} + \langle \psi_n^{KS} | \hat{\Sigma}(E_n^{QP}) - \hat{V}_{xc} | \psi_n^{KS} \rangle$$

# Back to basics of the classical world: Maxwell equations for the total field

- Maxwell equations:  $Q = -e$ ;  $n = \text{density}$

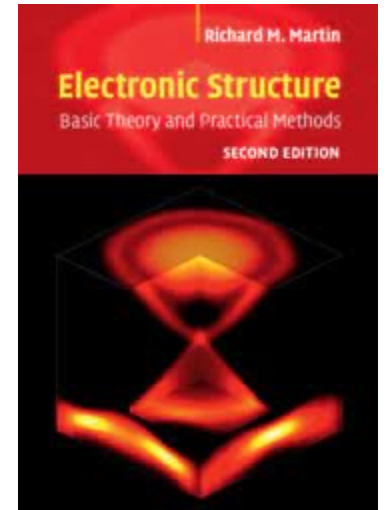
$$\nabla \cdot \mathbf{E} = 4\pi Qn$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E}(t) = -\frac{1}{c} \frac{d\mathbf{B}}{dt}$$

$$\nabla \times \mathbf{B}(t) = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{d\mathbf{E}}{dt}$$

$$\nabla \cdot \mathbf{j} = -Q \frac{dn}{dt}$$



- Internal and External charges and currents:

- $n = n_{\text{int}} + n_{\text{ext}}$ ;  $\mathbf{j} = \mathbf{j}_{\text{int}} + \mathbf{j}_{\text{ext}}$

- Polarization : defined to *within an additive constant* (one computes *polarization differences*)

$$\mathbf{P}(\mathbf{r}, t) = \int^t dt' \mathbf{j}_{\text{int}}(\mathbf{r}, t')$$

$$\nabla \cdot \mathbf{P}(\mathbf{r}, t) = -Qn_{\text{int}}(\mathbf{r}, t)$$

# Maxwell equations for the external field

- $\mathbf{D} = \mathbf{E} + 4\pi\mathbf{T}\mathbf{P}$   $\mathbf{D}$  = external field, **independent of the material**

$$\begin{aligned} \nabla \cdot \mathbf{D} &= 4\pi Q n_{\text{ext}} & \nabla \times \mathbf{E}(t) &= -\frac{1}{c} \frac{d\mathbf{B}}{dt} \\ \nabla \cdot \mathbf{B} &= 0 & \nabla \times \mathbf{B}(t) &= \frac{4\pi}{c} \mathbf{j}_{\text{ext}} + \frac{1}{c} \frac{d\mathbf{D}}{dt} \end{aligned}$$

- Relation between current and **total field** and density and **total field**

$$\mathbf{j}_{\text{int}}(\mathbf{r}, t) = \int d\mathbf{r}' \int^t \sigma(\mathbf{r}, \mathbf{r}', t - t') \mathbf{E}(\mathbf{r}', t')$$

$$\mathbf{j}_{\text{int}}(\mathbf{r}, \omega) = \int d\mathbf{r}' \sigma(\mathbf{r}, \mathbf{r}', \omega) \mathbf{E}(\mathbf{r}', \omega)$$

$$\mathbf{D}(\mathbf{r}, \omega) = \int d\mathbf{r}' \epsilon(\mathbf{r}, \mathbf{r}', \omega) \mathbf{E}(\mathbf{r}', \omega)$$

$$\mathbf{E}(\mathbf{r}, \omega) = \int d\mathbf{r}' \epsilon^{-1}(\mathbf{r}, \mathbf{r}', \omega) \mathbf{D}(\mathbf{r}', \omega)$$

- Response to the **total field**  $\mathbf{E}$   $\epsilon(\mathbf{r}, \mathbf{r}', \omega) = 1\delta(\mathbf{r} - \mathbf{r}') + \frac{4\pi i}{\omega} \sigma(\mathbf{r}, \mathbf{r}', \omega)$

- Response to the **external field**  $\mathbf{D}$   $\rightarrow \epsilon^{-1}(\mathbf{r}, \mathbf{r}', \omega)$

# Response in terms of scalar potentials

- $\mathbf{E} = -\nabla V(\mathbf{r})$  : field derived from potential  $\rightarrow$  in Fourier space:  $E(\mathbf{q}) = i \mathbf{q} V(\mathbf{q})$  is longitudinal ( $\parallel$  to  $\mathbf{q}$ )

$$\epsilon^{-1}(\mathbf{q}, \mathbf{q}', \omega) = \frac{\delta V_{\text{total}}^C(\mathbf{q}, \omega)}{\delta V_{\text{ext}}(\mathbf{q}', \omega)}$$

External potential

$$\epsilon(\mathbf{q}, \mathbf{q}', \omega) = \frac{\delta V_{\text{ext}}(\mathbf{q}, \omega)}{\delta V_{\text{total}}^C(\mathbf{q}', \omega)}$$

Coulomb potential

- How do we compute the dielectric response?
  - Derive an expression of the **direct and inverse dielectric response in terms** of single particle (Kohn-Sham) **electronic states  $\psi_i$**  from **density response functions**

# Static density response function

- Response of the electrons to a variation of the total potential at  $\mathbf{r}'$

$$\chi_n^0(\mathbf{r}, \mathbf{r}') = \frac{\delta n(\mathbf{r})}{\delta V_{\text{eff}}(\mathbf{r}')} = 2 \sum_{i=1}^{\text{occ}} \sum_j^{\text{empty}} \frac{\psi_i^*(\mathbf{r})\psi_j(\mathbf{r})\psi_j^*(\mathbf{r}')\psi_i(\mathbf{r}')}{\epsilon_i - \epsilon_j}$$

Definition

Perturbation theory

$$\chi_n^0(\mathbf{r}, \mathbf{r}') = \sum_{i=1}^{\text{occ}} \psi_i^*(\mathbf{r}) G_0^i(\mathbf{r}, \mathbf{r}') \psi_i(\mathbf{r}')$$

Independent particle Green function

$$G_0^i(\mathbf{r}, \mathbf{r}') = \sum_{j \neq i}^{\infty} \frac{\psi_j(\mathbf{r})\psi_j^*(\mathbf{r}')}{\epsilon_i - \epsilon_j}$$



# Response in terms of scalar potentials

- $\mathbf{E} = -\nabla V(\mathbf{r})$  : field derived from potential  $\rightarrow$  in Fourier space:  $E(\mathbf{q}) = i \mathbf{q} V(\mathbf{q})$  is longitudinal ( $\parallel$  to  $\mathbf{q}$ )

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External potential

$$\epsilon(\mathbf{q}, \mathbf{q}', \omega) = \frac{\delta V_{\text{ext}}(\mathbf{q}, \omega)}{\delta V_{\text{total}}^C(\mathbf{q}', \omega)}$$

Coulomb potential

- How do we compute the dielectric response?

$$\epsilon^{-1} = 1 + \frac{V_C \chi^0}{1 - (V_C + f_{xc}) \chi^0}$$

- Derive an expression of the **direct and inverse dielectric response in terms** of single particle (Kohn-Sham) **electronic states  $\psi_i$**  from **density response functions**

# Calculation of dielectric matrices

Within the RPA approximation ( $\mathbf{f}_{xc} = 0$ )

$$\epsilon_{\mathbf{G},\mathbf{G}'}(\mathbf{q}) = \delta_{\mathbf{G},\mathbf{G}'} - \frac{4\pi e^2}{|\mathbf{q} + \mathbf{G}|^2} \frac{4}{N_k \Omega} \sum_{\mathbf{k},v,c} \frac{\langle v, \mathbf{k} | e^{-i(\mathbf{q} + \mathbf{G}) \cdot \mathbf{r}} | c, \mathbf{k} + \mathbf{q} \rangle \langle c, \mathbf{k} + \mathbf{q} | e^{i(\mathbf{q} + \mathbf{G}') \cdot \mathbf{r}'} | v, \mathbf{k} \rangle}{E_{v,\mathbf{k}} - E_{c,\mathbf{k} + \mathbf{q}}}$$

Similarity transformation to a Hermitian matrix:

$$\tilde{\epsilon}_{\mathbf{G},\mathbf{G}'}(\mathbf{q}) = \frac{|\mathbf{q} + \mathbf{G}|}{|\mathbf{q} + \mathbf{G}'|} \epsilon_{\mathbf{G},\mathbf{G}'}(\mathbf{q})$$

Eigenvalues of the dielectric matrix are real and greater than or equal to 1

**Direct, straightforward calculation of dielectric matrices is prohibitive for large systems**

# Spectral decomposition

- Represent polarizability by its **eigenvalue decomposition** and **truncate sum** over eigenvalues to an *appropriate, small number*

$$\tilde{\chi}_0 = \sum_{i=1}^N \tilde{\phi}_i \lambda_i \tilde{\phi}_i^H \xrightarrow{\text{RPA}} \tilde{\chi} = \sum_{i=1}^N \tilde{\phi}_i \frac{\lambda_i}{1 - \lambda_i} \tilde{\phi}_i^H$$

- Once this eigenvalue decomposition is known, computing  $\epsilon$  is trivial

$$\tilde{\epsilon} = \sum_{i=1}^N \tilde{\phi}_i (1 - \lambda_i) \tilde{\phi}_i^H \qquad \tilde{\epsilon}^{-1} = \sum_{i=1}^N \tilde{\phi}_i \left( \frac{\lambda_i}{1 - \lambda_i} + 1 \right) \tilde{\phi}_i^H$$

H.Wilson, F.Gygi and G.Galli, PRB 2008; H.Wilson, D.Lu, F.Gygi & GG, PRB 2009

- Compute eigenvalues and eigenvectors using **Density Functional Perturbation Theory**\* (DFPT)  $\rightarrow$  avoid costly calculation of empty single particle states

(\* S. Baroni, et al., Rev. Mod. Phys., 73:515, 2001.

# Calculations of dielectric matrices: spectral decomposition & DFPT

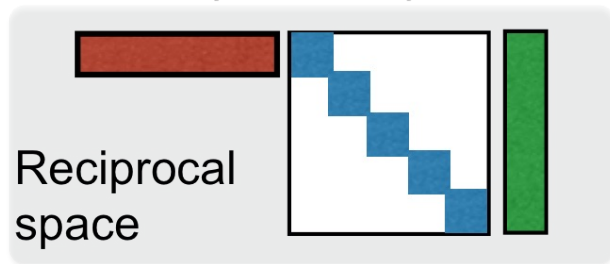
$$\tilde{\epsilon}^{-1} = \sum_{i=1}^{\text{Neig}} \tilde{\phi}_i \left( \frac{\lambda_i}{1 - \lambda_i} + 1 \right) \tilde{\phi}_i^H$$

- Calculation of **empty electronic states**, calculation and storage of **full dielectric matrix** and **inversion** of  $\tilde{\epsilon}$  are **avoided**
- **Scaling**:  $N_{\text{eig}} N_{\text{pw}} N_v^2$  (instead of  $N_{\text{pw}}^2 N_v N_c$ )
- Efficient evaluation of  $\tilde{\epsilon}^{-1}$  at different  $\mathbf{q}$  points and at different MD steps is possible

# Low rank decomposition of the screened Coulomb interaction $W$

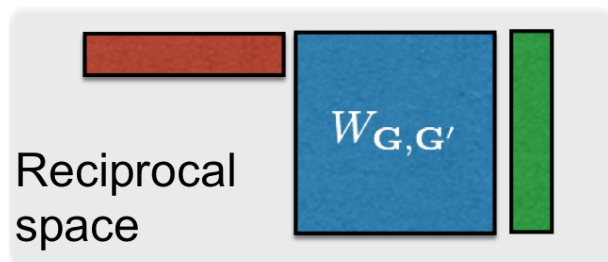
In **Hartree-Fock**

$$\langle \psi_i \psi_j | \frac{1}{|\mathbf{r} - \mathbf{r}'|} | \psi_k \psi_l \rangle$$



In **GW**

$$\langle \psi_i \psi_j | W(\mathbf{r}, \mathbf{r}') | \psi_k \psi_l \rangle$$

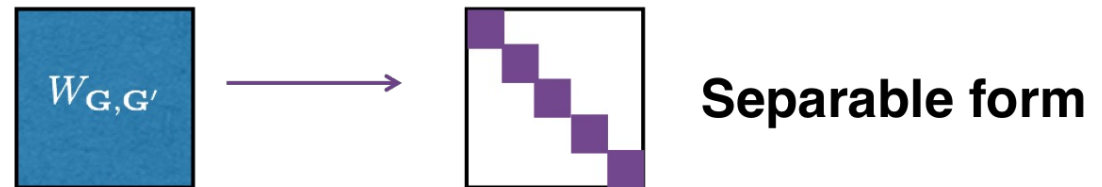


Example : 64 water molecules

<b>Direct space</b>	size $\sim (250)^3 \times (250)^3$ Difficult to truncate
<b>Reciprocal space</b>	size $\sim (1'000'000) \times (1'000'000)$ Could be truncated, full matrix
<b>Eigenpotential space</b>	size $\sim (1'000) \times (1'000)$

$$W = \sum_{\alpha} |\alpha\rangle \lambda_{\alpha} \langle \alpha|$$

**Low-rank decomposition**





# Summary of GW algorithm

- Iterative diagonalization of the dielectric matrix (+) →
- Low rank decomposition of  $W$
- DFPT (\*) based projection techniques to compute  $G$
- Eigenpotentials of  $\tilde{\epsilon}$  as basis set also at finite frequency (++)
- Lanczos algorithm to compute frequency dependence of dielectric matrix in parallel
- Contour deformation technique for frequency integration (&)

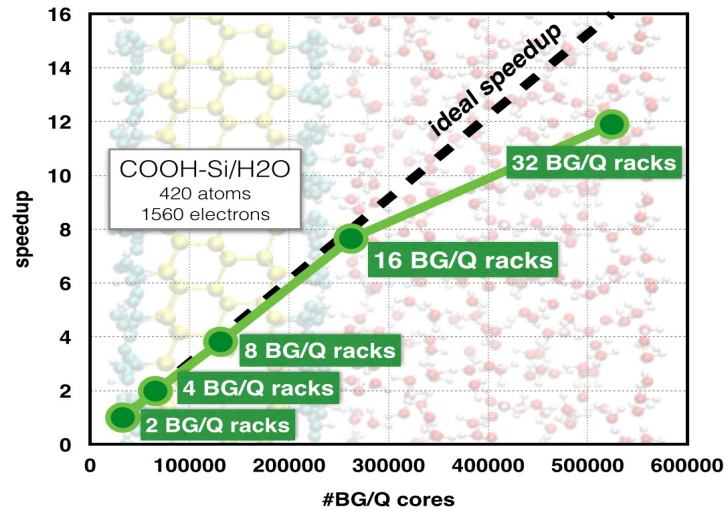
(\*) S. Baroni, et al., Rev. Mod. Phys., 73:515, 2001.

(+) H. Wilson, F. Gygi, and G. G., PRB 2008; H.Wilson, D.Lu, F.Gygi and G.G., Phys.Rev.B 2009

(++) H. V. Nguyen, T.A. Pham, D.Rocca and GG Phys. Rev. B (R) 2012; T.A.Pham, H,V.Nguyen, D.Rocca and GG, Phys.Rev.B 2013

(&) M.Govoni & GG, J. Chem. Theory Comput., (2015) & J.Chem.Theory Comp. (2018).

# Implementation of GW algorithm



## Range of applicability

Ordered and disordered **solids**,  
defective materials, **liquids**,  
molecular crystals,  
**nanostructures**, interfaces



Govoni&GG, JCTC 2015, JCTC  
2018

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[www.west-code.org](http://www.west-code.org) scalable to > 500,000 cores

- Eliminated summations over **empty states** using DFPT
- W made **separable** using the eigenvectors of the dielectric matrix as basis set; number of eigenpotentials controls the **accuracy** of the method.
- Greatly **reduced pre-factors** of  $O(N^4)$  scaling

# Absorption of light: solving the Bethe Salpeter equation (BSE)

Quantum Liouville equation

$$i\frac{d\hat{\rho}(t)}{dt} = [\hat{H}(t), \hat{\rho}(t)]$$

$$\hat{H}(t)\phi(\mathbf{r}, t) = \left[ -\frac{1}{2}\nabla^2 + v_H(\mathbf{r}, t) + v_{ext}(\mathbf{r}, t) \right] \phi(\mathbf{r}, t) + \int \Sigma(\mathbf{r}, \mathbf{r}', t) \phi(\mathbf{r}', t) d\mathbf{r}'$$

$$\Sigma_{COH}(\mathbf{r}, \mathbf{r}') = \frac{1}{2}\delta(\mathbf{r} - \mathbf{r}')W_p(\mathbf{r}', \mathbf{r}) \quad \text{BSE}$$

$$\Sigma_{SEX}(\mathbf{r}, \mathbf{r}', t) = -\sum_v \phi_v(\mathbf{r}, t)\phi_v^*(\mathbf{r}', t)W(\mathbf{r}', \mathbf{r})$$

Screened Coulomb interaction

- The quantum Liouville equation is solved within linear response theory
- Explicit calculation of empty electronic states is avoided by using iterative diag. of  $\epsilon$

D. Rocca, D. Lu, and G. Galli, *JCP* (2010)

D. Rocca, Y. Ping, R. Gebauer, and G. Galli, *PRB* (2012)

D. Rocca, R. Gebauer, Y. Saad, and S. Baroni, *JCP* (2008)

B. Walker, R. Gebauer, A. M. Saitta, and S. Baroni, *PRL* (2006)

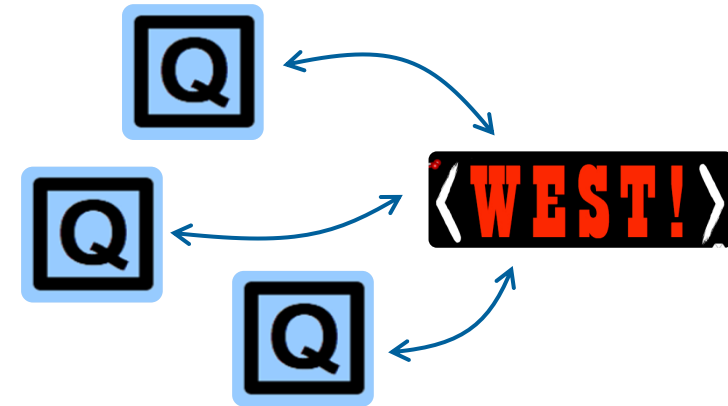
# Addressing existing drawbacks

Develop efficient and scalable algorithms to go **beyond the random phase approximation (RPA)**, which are 'easily' applicable to **hybrid DFT** wavefunctions and lead to **improved efficiency**

**Strategy:**

$$\text{RPA: } f_{\text{xc}} = \frac{\delta v_{\text{xc}}[n]}{\delta n} \approx 0$$

- A **finite field algorithm** to compute the density-density response function
- DFT calculations performed by **Qbox** exploiting:
  - Fast hybrid functional calculations with controllable accuracy
  - Client-server mode



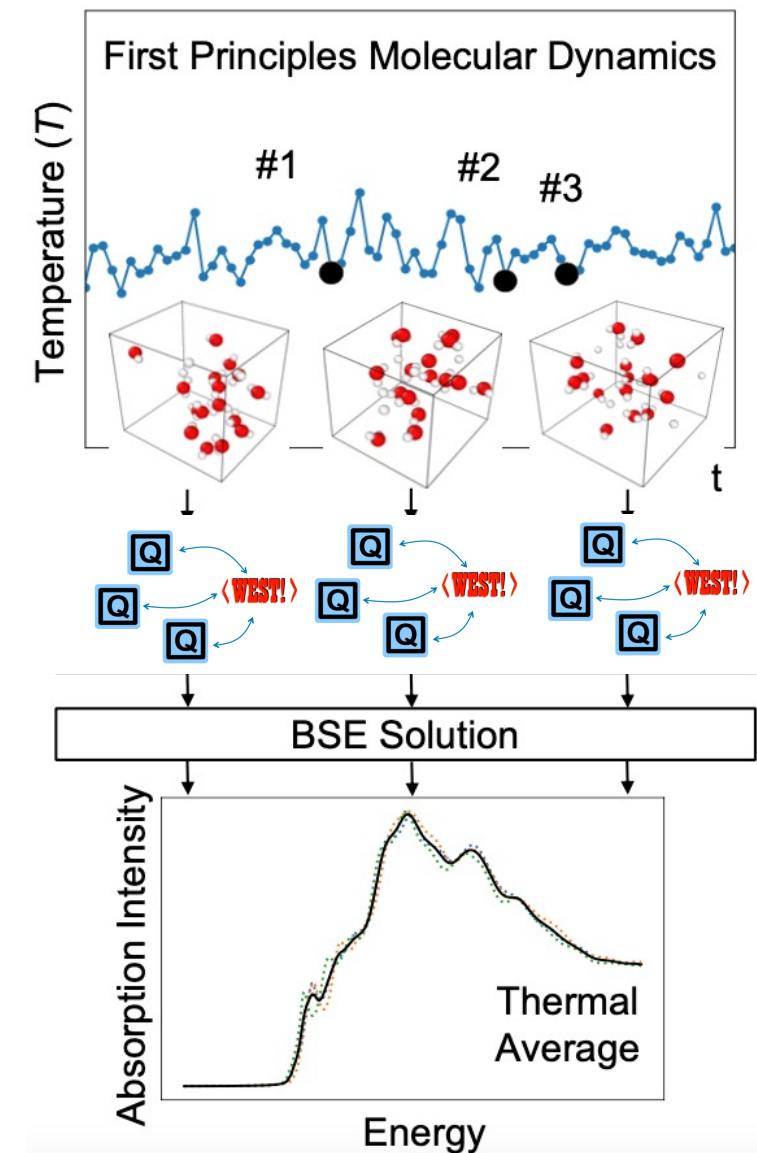
# Finite temperature optical spectra

We developed an efficient solver of the Bethe-Salpeter Equation (BSE), based on a finite field approach (unique to WEST)\*:

- **WEST-Qbox coupling** to avoid direct calculation of dielectric matrices and overcome commonly used approximations (e.g., Random Phase Approximation)
- **Reduced workload** by harnessing orbital localization w/ the recursive bisection method (unique to Qbox)

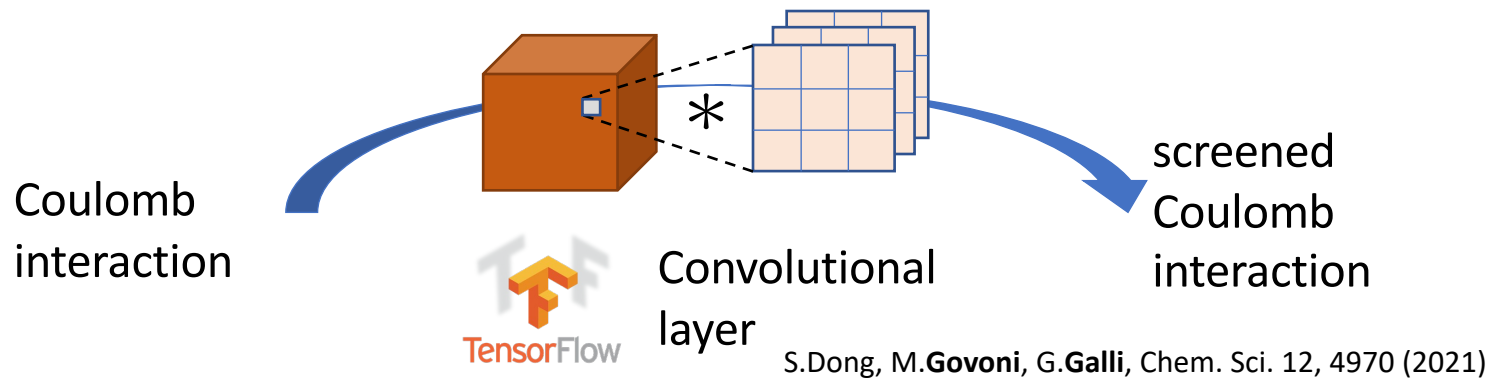
We computed **optical spectra** at finite  $T$  (e.g. liquid water and ice), where BSE is solved for several snapshots extracted from MD trajectories.

\*L. Nguyen, H.Ma, M.Govoni, F.Gygi, G.Galli, *PRL* 122, 237402 (2019)

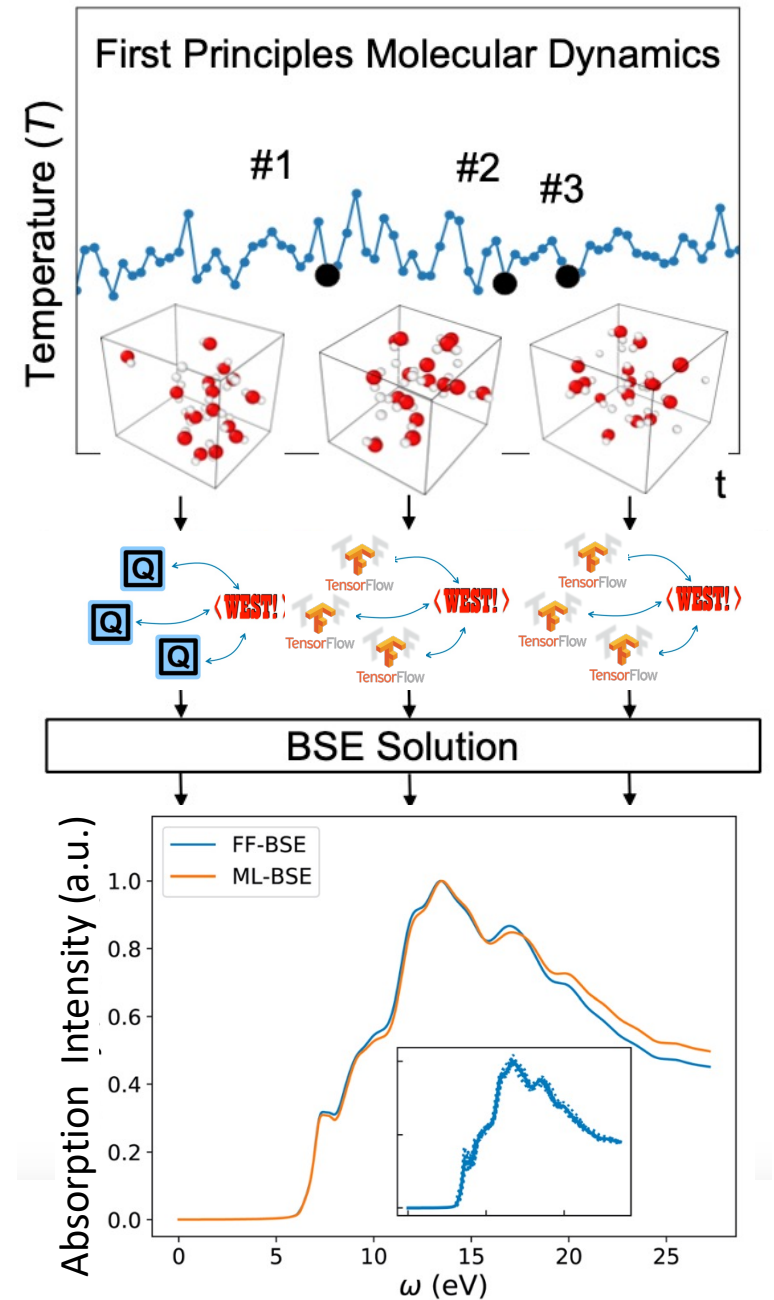


# Data driven approach to obtain the dielectric screening

- Identified a **machine learning** protocol to avoid redundant calculations of the dielectric screening
- “Learn” on-the-fly and use a data-driven model for the dielectric response → speedup 100x

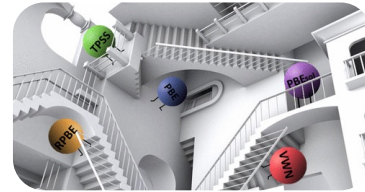


- A strategy to represent physical quantities in electronic structure theory using ML
- Future work: use similar concept to derive improved density functionals for interfaces**



# Some open questions

- 'Everything' relies on 'DFT ground states': which DFT?
  - Which functional for interfaces?



- How do we optimize geometries of excited states and deal with conical intersections?
- Can we 'recycle' quantities that are computed many times in electronic structure calculations? Can we 'learn' them and 'extrapolate' them? (e.g. dielectric matrices) ?
- What's the future of Many-Body-Perturbation-Theory?
  - Self-consistency? 'Additional' diagrams ? Use instead quantum chemistry methods for excited states also in the solid state? Role of Quantum Monte Carlo (QMC)?